

## Matrices pour Pile ou Face

> *with(LinearAlgebra) :*

Les lignes qui se terminent par : sont exécutées sans écho à la ligne du dessous

### ▼ Avec PPF et PFF

> *ip := Matrix(6, 6, [1, -2, 2, 0, 0, 0, 1, 1, 2, 0, 3, 0, 0, 0, -1, 2, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, -1, -3, 1, -5, 1])*

$$ip := \begin{bmatrix} 1 & -2 & 2 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -3 & 1 & -5 & 1 \end{bmatrix} \quad (1.1)$$

> *p := MatrixInverse(ip);*

$$p := \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{1}{4} & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} & -\frac{1}{4} & 0 \\ \frac{1}{8} & 0 & \frac{1}{2} & \frac{1}{4} & -\frac{1}{8} & 0 \\ -\frac{1}{12} & \frac{1}{3} & 0 & -\frac{1}{2} & -\frac{1}{4} & 0 \\ -\frac{1}{24} & \frac{5}{3} & -\frac{1}{2} & -\frac{3}{4} & -\frac{5}{8} & 1 \end{bmatrix} \quad (1.2)$$

> *J := Matrix(6, 6, [-1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2]);*

$$J := \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} \quad (1.3)$$

> *MMM := MatrixMatrixMultiply:*

```
> M6 := MMM(MMM(p, J), ip);
```

$$M6 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \end{bmatrix}$$

(1.4)

```
> JordanForm(M6)
```

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

(1.5)

### ▼ Avec PPPFFF et FPPPPP :

```
> M := Matrix(13, 13, 0) :
```

```
> M[2, 1] := 1 : M[3, 2] := 1 : M[4, 3] := 1 : M[4, 4] := 1 :
```

```
> M[5, 4] := 1 : M[5, 11] := 1 : M[5, 12] := 1 : M[6, 5] := 1 : M[7, 6] := 1 : M[7, 7] := 2 :
```

```
> M[8, 1] := 1 : M[8, 8] := 1 : M[8, 9] := 2 : M[8, 10] := 1 : M[9, 5] := 1 : M[9, 6] := 1 : M[9, 8] := 1 : M[10, 9] := 1 : M[11, 10] := 1 : M[12, 11] := 1 : M[13, 12] := 1 : M[13, 13] := 2 :
```

```
> evalm(M);
```

(2.1)

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0
 \end{bmatrix}
 \tag{2.1}$$

> *CharacteristicPolynomial(M, X)*  

$$X^3 (-1 + X) (1 + X^7 - X^6 - 2 X^5 - X^4 - X^3 - X^2 + X) (-2 + X)^2$$
(2.2)

> *evalf(allvalues(RootOf(1 + X^7 - X^6 - 2 X^5 - X^4 - X^3 - X^2 + X)))*  
0.7401177052, 2.213897801, 0.1853455379 + 0.9590637684 I, -0.7987037925  
+ 0.4914449776 I, -0.7272989971, -0.7987037925 - 0.4914449776 I, 0.1853455379  
- 0.9590637684 I  
(2.3)

> *map(abs, [%])*  
[0.7401177052, 2.213897801, 0.9768092343, 0.9377877767, 0.7272989971,  
0.9377877767, 0.9768092343 ]  
(2.4)

> *evalm(MatrixPower(M, 100))*;  
[[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],  
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],  
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],  
[1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],  
[587608635742846539908345251190989, 153503217725634575874331971836056,  
339840436186110781145458685963604, 752371994400735594977665509164397,  
913302709619890406047488505829969, 629129793696918732239234761928485, 0,  
1147402248858772366632194388461425, 1392829066878473445659194058312731,  
788776710766971510023032687838495, 598868776675101019103333537328341,  
412531558214624813832206823200793, 0],  
[265418139648632320170468931852117, 69336180580992905515385624658049,  
153503217725634575874331971836056, 339840436186110781145458685963604,

(2.5)

412531558214624813832206823200793, 284172915922971673808253743901484, 0,  
518272455161853634392959626532940, 629129793696918732239234761928485,  
356284156554766176873274805246851, 270504255605117875630073061305555,  
186337218460476205271126714127548, 0],  
[1240813567807329274525569364864120, 324206355222553461398435976251535,  
717748891026099828312257577161119, 1589000999777834232498847126158294,  
1928841435963945013644305812121898, 1329103054939923798904687135582827,  
1267650600228229401496703205376, 2422838920040737407823171685328822,  
2941111375202591042216131311861762, 1665674704020626001025154014994365,  
1264794644555280771100411149906759, 871252108751734404186589548997175, 0  
],  
[8870950871640926188255785386928769, 2317391917430618330901670850280051,  
5130468870310016698559019253735841, 11358333750643280689773990535745578,  
13787856364180769923449008269845158, 9497783307278256439773519460898630,  
0, 17321986711051153526498259133244684,  
21027121579403759747599519912384029,  
11907924806214884683752713211499180, 9040941833212662358872319685465527,  
6227864880333263991214971282009737, 0],  
[4392243238218348018587669175896069, 1147402248858772366632194388461425,  
2540231315737245812291388446774156, 5623812524221762055578809911535501,  
6826733657008365010318304819338078, 4702604615117138297507695632147321,  
0, 8576575398173632681702405181510450,  
10411086016898146845821007966728599, 5895929643631886562093138369604345,  
4476410275362989688946615523074076, 3083581208484516243287421464761345, 0  
],  
[1983941280404149631968806244803828, 518272455161853634392959626532940,  
1147402248858772366632194388461425, 2540231315737245812291388446774156,  
3083581208484516243287421464761345, 2124129041891226712810609187190757,  
0, 3873970783056494384194709549363129,  
4702604615117138297507695632147321, 2663144450785158077431588868002341,  
2021958860575392177898428820241216, 1392829066878473445659194058312731, 0  
],  
[896130471519206961468040887933931, 234099539238881960584705882631456,  
518272455161853634392959626532940, 1147402248858772366632194388461425,

1392829066878473445659194058312731, 959452166593289530476812277570588, 0,  
 1749841741165267671384100362172372, 2124129041891226712810609187190757,  
 1202921132786602954739494907802577, 913302709619890406047488505829969,  
 629129793696918732239234761928485, 0],  
 [404774995065199981038485472363549, 105740896947228820560752803332147,  
 234099539238881960584705882631456, 518272455161853634392959626532940,  
 629129793696918732239234761928485, 433376900285183915182381780742143, 0,  
 790389574571978140907288084601784, 959452166593289530476812277570588,  
 543349892747270430996033017987189, 412531558214624813832206823200793,  
 284172915922971673808253743901484, 0],  
 [1892255244959733256212629338029161, 494314396726849451529240049008268,  
 1094369690400927723619232901348683, 2422838920040737407823171685328822,  
 2941111375202591042216131311861762, 2025991881272305633829705201636622,  
 0, 3694971088257817041934504099413603,  
 4485360662829795182841792184015387, 2540231315737245812291388446774156,  
 1928841435963945013644305812121898, 1329103054939923798904687135582827,  
 1267650600228229401496703205376]]

>  $e1100 := \text{MatrixPower}(M, 100)[7, 1];$   
 $e1100 := 1240813567807329274525569364864120$  (2.6)

>  $e5100 := \text{MatrixPower}(M, 100)[13, 1];$   
 $e5100 := 1892255244959733256212629338029161$  (2.7)

>  $\text{evalf}\left(\frac{e5100}{e1100}\right)$   
 1.525011729 (2.8)

## ▼ Avec FPPPPP et FFPPFP

curiosité : j'avais par erreur mis  $M(2,11)=1$  au lieu de  $M(8,11)=1$  (le successeur de FFPPF par F est FF et non F)

il en résultait une matrice dont les termes  $M^n(1,7)$  (x gagne) et  $M^n(1,12)$  (y gagne) étaient toujours égaux (?????)

>  $M := \text{Matrix}(12, 12, 0) :$

>  $M[1, 1] := 1 :$

$M[2, 1] := 1 : M[2, 3] := 1 : M[2, 4] := 1 : M[2, 5] := 1 : M[2, 6] := 1 : M[2, 9] := 1 :$

$M[3, 2] := 1 :$

$M[4, 3] := 1 :$

$M[5, 4] := 1 : M[5, 10] := 1 :$

$M[6, 5] := 1 :$

$M[7, 6] := 1 : M[7, 7] := 2 :$

```

M[8, 2] := 1 : M[8, 8] := 1 : M[8, 11] := 1 :
M[9, 8] := 1 :
M[10, 9] := 1 :
M[11, 10] := 1 :
M[12, 11] := 1 : M[12, 12] := 2 :

```

```
> evalm(M);
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

(3.1)

```
> CharacteristicPolynomial(M, X)
```

$$(-1 + X) (1 + X^9 - X^8 - X^7 - X^6 - X^5 - X^4 + X^3 + X^2 + X) (-2 + X)^2$$

(3.2)

```
> evalf(allvalues(RootOf(X^10 - 5 X^9 + 6 X^8 + 3 X^7 - 5 X^6 - 5 X^5 + 6 X^4 + X^3 - 2 X^2 - 3 X)))
```

```
0., 1.479082265, 1.898543272, 2.554851992, 0.7447091542 + 0.7076692174 I,
```

(3.3)

```
-0.4242948788 + 0.5406247113 I, -0.7866530399 + 0.4691267158 I, -0.7866530399
```

```
-0.4691267158 I, -0.4242948788 - 0.5406247113 I, 0.7447091542 - 0.7076692174 I
```

```
> map(abs, [%])
```

```
[0., 1.479082265, 1.898543272, 2.554851992, 1.027320517, 0.6872417498, 0.9159164158,
```

(3.4)

```
0.9159164158, 0.6872417498, 1.027320517]
```

```
> Mp100 := MatrixPower(M, 100) :
```

```
> Mp100[7, 1]
```

```
594734857996916564604450886255
```

(3.5)

```
> Mp100[12, 1]
```

```
634170771924622573836007562384
```

(3.6)

```
> evalf( (Mp100[12, 1] / Mp100[7, 1]) )
```

```
1.066308395
```

(3.7)

```
> Mp10 := MatrixPower(M, 10) :
```

```
> evalm(Mp10)
```

$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 184 & 171 & 170 & 158 & 134 & 88 & 0 & 158 & 146 & 112 & 82 & 0 \\
 96 & 88 & 89 & 82 & 70 & 46 & 0 & 82 & 76 & 58 & 42 & 0 \\
 50 & 46 & 46 & 43 & 36 & 24 & 0 & 42 & 40 & 30 & 22 & 0 \\
 58 & 54 & 54 & 50 & 43 & 28 & 0 & 50 & 46 & 36 & 26 & 0 \\
 30 & 28 & 28 & 26 & 22 & 15 & 0 & 26 & 24 & 18 & 14 & 0 \\
 79 & 109 & 142 & 203 & 323 & 559 & 1024 & 104 & 130 & 175 & 45 & 0 \\
 232 & 216 & 216 & 200 & 170 & 112 & 0 & 201 & 186 & 142 & 104 & 0 \\
 120 & 112 & 112 & 104 & 88 & 58 & 0 & 104 & 97 & 74 & 54 & 0 \\
 62 & 58 & 58 & 54 & 46 & 30 & 0 & 54 & 50 & 39 & 28 & 0 \\
 32 & 30 & 30 & 28 & 24 & 16 & 0 & 28 & 26 & 20 & 15 & 0 \\
 80 & 112 & 79 & 76 & 68 & 48 & 0 & 175 & 203 & 320 & 592 & 1024
 \end{bmatrix}$$

(3.8)

### Avec FFPPFP et PPPFFF

```

> M := Matrix(13, 13, 0) :
> M[2, 1] := 1 : M[2, 4] := 1 : M[2, 8] := 1 : M[2, 9] := 1 :
  M[3, 2] := 1 : M[3, 3] := 1 : M[3, 6] := 1 :
  M[4, 3] := 1 : M[4, 12] := 1 :
  M[5, 4] := 1 :
  M[6, 5] := 1 :
  M[7, 6] := 1 : M[7, 7] := 2 :
  M[8, 1] := 1 : M[8, 2] := 1 : M[8, 11] := 1 :
  M[9, 8] := 1 :
  M[10, 5] := 1 : M[10, 9] := 1 : M[10, 10] := 1 :
  M[11, 10] := 1 :
  M[12, 11] := 1 :
  M[13, 12] := 1 : M[13, 13] := 2 :
> evalm(M);

```

(4.1)

$$\begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2
 \end{bmatrix}
 \tag{4.1}$$

> *CharacteristicPolynomial(M, X)*  

$$X(X^{10} - 2X^9 + 2X^4 - X)(-2 + X)^2$$
(4.2)

> *evalf(solve(X<sup>10</sup> - 2X<sup>9</sup> + 2X<sup>4</sup> - X))*  
 0., 1., 0.916143459230787, 1.93061307038875, 0.265698715810085  
 + 1.02303748806730 I, -0.344148376567828 + 0.654872079220998 I,  
 -0.844928604052024 + 0.459094706146803 I, -0.844928604052024  
 - 0.459094706146803 I, -0.344148376567828 - 0.654872079220998 I,  
 0.265698715810085 - 1.02303748806730 I

(4.3)

> *map(abs, [%])*  
 [0., 1., 0.916143459230787, 1.93061307038875, 1.05697753503761, 0.739794258721642,  
 0.961598822356455, 0.961598822356455, 0.739794258721642, 1.05697753503761]

(4.4)

> *Mp100 := MatrixPower(M, 100) :*  
 > *Mp100[7, 1]*  
 591253824509439906038468786991

(4.5)

> *Mp100[13, 1]*  
 631775935492282389438923129480

(4.6)

>  $\frac{Mp100[13, 1]}{Mp100[7, 1]}$   

$$\frac{631775935492282389438923129480}{591253824509439906038468786991}$$
(4.7)

> *evalf(%)*  
 1.068535897

(4.8)